- ENGINEERING MEDIEVAL CATHEDRALS

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piers. The Chapter was pusillanimous, sought further advice, and only two years later finally decided, on 17 April 1573, to put the work in hand. Thirteen days later, on Ascension Day, 30 April, the tower fell; Desjardins, Pihan, and Leblond all say that the two "open" crossing piers failed first. The clergy and people had just left the cathedral in procession; only three people were left inside, and all three escaped. The Chapter decided, in 1577, to celebrate annually on 30 April the signal protection that the faithful of Beauvais had been afforded.¹ Otherwise, however, the Chapter lost heart at this stage. By 1578 all necessary repairs had been made (but the tower had not been replaced); equally, all the money set aside for the nave had been spent. There were sporadic attempts to complete the cathedral, but in 1605 the decision was taken to consolidate the existing work, and Beauvais became what it is today, a choir and transept without a nave. "Lé temps n'était plus à bâtir des cathédrales. Les écoles d'architectes, de sculpteurs, de verriers, de peintres, que leur construction avait fait surgir, se mouraient de toutes parts."2

Branner has pointed out³ that, despite all that has been written about the colossal dimensions of Beauvais, they are not much greater in fact than those of the great cathedrals of the first half of the thirteenth century. The centre-line width between main piers of the choir of Beauvais is 15.0 m., almost exactly that of Bourges, Chartres, Amiens and Cologne, and slightly more than Reims; the total width of the choir (about 42 m.) is about the same as Bourges and less than all the others, so that the width of the side aisles is, significantly, less than the others. Only the height of the vault, 48 m., is greater than the others, and Cologne has a height of 46 m. As for the spacing of the piers in the axial direction, the three original bays of the choir at Beauvais varied from about 8 m. to 9 m.,4 slightly smaller than the largest bay at Amiens, and almost exactly the same as at Reims and Cologne.

Whatever reasons can be given for the fall of 1284, therefore, they cannot be tied to any unusual daring on the part of the designer. The question to be asked is, rather: if Amiens and Cologne stood, then why not Beauvais? Cologne took even longer to build than Beauvais, the choir being finished only in 1322 and the nave not yet started. Amiens was constructed between 1220 and 1288, spanning in time the first phase of Beauvais. Thus the evidence is that Gothic technique was efficient at least until the end of the thirteenth century; or, at any rate, that a building *designed* in the first half of that century could be built in the second half.

Benouville, reporting on the structure of Beauvais in 1891, finds that there is a difference in the quality of the work above and below the triforium; below "l'appareil . . . est très soigné, très regulier," but above, less so. Fifty years is, of course, too long for a single man to have been in charge, and Benouville concludes that the final phase was directed by someone less skilled than a maître d'oeuvre, but working to existing drawings.⁵ Branner has made a detailed study of the chronology, and finds

¹ Legends have grown up about this spectacular collapse. For example, no one would undertake the dangerous job of clearing the rubble in the partially destroyed building. Finally, four months later, a condemned criminal was offered his life if he would demolish the ruins. He had only just started when his footing gave way, and he fell, but managed to catch hold of a rope hanging from the roof beams, and so climbed to safety. "La corde qui devait être le supplice de ce misérable fut son salut." (Desjardins, op. cit., p. 99).

 ² Desjardins (op. cit.), p. 110.
³ Robert Branner, "Le maître de la Cathédrale de Beauvais," Art de France, II, Paris (1962), 77-92.
⁴ E. E. Viollet-le-Duc, in his Dictionnaire raisonné de l'architecture Francaise du XI^e au XVI^e siècle, 10 vol., ¹ E. E. Violet-le-Duc, in fis Dictionnaire raisonne de l'architecture riancaise du AI et a AI et a le construire francaise du AI et al. (article Proportion), comments on the bay spacing; to reduce the lateral thrust on the crossing piers, the adjacent bay was reduced in width. "L'architecte... sent que les grandes archivoltes ... vont exercer une poussée active sur la première pile ... du choeur, qui n'est plus etrésillonée à la hauteur de ces archivoltes. D'abord il augmente la section de cette pile, puis il diminue l'écartement de la première travée..." ^s L. Benouville, "Étude sur la Cathédrale de Beauvais," *Encyclopédie d'architecture*, 4th series, 4, Paris (1891–2), 52–54, 60–62, 68–70. "... nous en conclurons que, lorsqu'on a achevé Beauvais, le chantier n'était pas dirigé par celui qui l'avait commencé. Les plans existaient; un nouveau maître d'oeuvre ne fut pas appelé, ce fut un sous-

ordre qui fut chargé de terminer les travaux."

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that the work was taken up to triforium level between 1225 and 1245 under the first maître. There was an interim period, 1245–50, when a small amount was done under a second maître, and from 1250 on the work was carried on under the direction of a third maître. Whether this third master was an actual maître d'oeuvre or not, Branner believes that he constructed the high vaults higher than was intended by the first master, and created the famous intermediate buttresses in "porte-à-faux," although there seems to be no evidence that this construction was not intended by the first master. However this may be, in 1272 there existed at Beauvais vaults standing 48 m. from the ground; in 1284 these vaults fell. Twelve years is a long time for a masonry structure to tremble, trying to decide whether its main piers are too widely spaced (too wide for what?); similarly if, as Leblond says, the external buttresses failed, why did they not fail immediately? (Benouville believes¹ that the crossing tower was imprudently started late in the thirteenth century, but this seems to be a conflation with the story of the sixteenth-century collapse.)

A Gothic cathedral stands by virtue of a more or less delicate <u>balance of forces</u>; the <u>vaults</u> thrust out, and these thrusts are taken out and down through the <u>flying buttresses</u> to the main <u>buttresses</u>, and so to the <u>ground</u>. Certainly Beauvais is high, and the balance of forces must therefore be more rather than less critical; if, however, a system of thrusts can be found which indicates that, at least originally, the structure was safe, then the lower bound theorem of limit analysis² states that, under normal circumstances, the structure will continue to be safe.

Indeed, the structural analysis given below is almost not needed. The fact that Beauvais did stand for 12 years is ample experimental evidence that it was possible to achieve equilibrium between thrust and counter-thrust. Further, it can be shown, by the use of the same theorem of limit analysis, that any small shifts in the structure (for example, the settlement of a main pier) cannot of themselves promote collapse of the structure, providing the overall geometry is not significantly changed. Thus the fact that mediaeval mortar was slow drying, and liable to shrinkage over years or decades, would be significant only if the structure were in such delicate equilibrium that it would have collapsed in any case under any slight live loading, e.g. wind, or if such shrinkage could cause some secondary failure of an important structural component, and hence trigger off a catastrophic collapse. In fact the analysis given below indicates that Beauvais in its original state was comfortably in equilibrium. As a means of determining the cause of the vault collapse of 1284, therefore, the overall structural analysis is *a posteriori* irrelevant. However, the positive conclusion may be drawn from the analysis that collapse must be attributed either to some essentially trivial, but far-reaching cause, or to an unforeseen event (e.g. an earthquake). In the absence of any record of such a natural catastrophe, the reason for the collapse must be sought in some detail of the construction rather than in a major fault of the design.

STRUCTURAL ANALYSIS AND COMPARISON

The plan in Fig. 1 is Viollet-le-Duc's reconstruction³ of the original design of Beauvais, together with his idea of how the nave might have been built. Even if the number of nave bays is indeterminate, Viollet-de-Duc's plan shows something very close to what the first master must have had in mind; five typical bays are drawn, and each of these would have had the same structure. In a sense, these typical bays contain the structural (as well as the visual) essence of Gothic; the other portions of the complete structure, the chevet, the transept, even the towers are developed from the structure of the nave bays. In fact, of course, an actual cathedral, built under several masters, often altered in design

¹ Idem., "A la fin du treizième siècle on éleva imprudemment sur la croisée du transept, et avant d'avoir monté la nef, le clocher central."

² See note 2 on p. 16.

³ Viollet-le-Duc, op. cit., 2, 334, Fig. 22 (article Cathédrale.)

as it was built, so that the nave and choir are different in structure; in the case of Beauvais, only one *typical* bay was ever completed, i.e. the second bay east of the crossing (B in Fig. 1). The next bay east (A) is already part of the chevet; the previous bay (C) is part of the transept. In fact this single bay is structurally the most critical of the choir, receiving no additional buttressing from either the chevet or the transept, and it is this bay that Benouville analysed, and which will be discussed here. Corroyer's cross-section¹ of the "typical" bay is shown in Fig. 2, and Benouvilles' "coupe restaurée" in Fig. 3. Comparing these two figures, it will be seen that the present structure has two extra flying buttresses, added in the sixteenth century; these will be removed from the analysis, which will deal, as far as possible, with the completed choir of 1272.

Benouville's analysis consists in the drawing of force polygons for a dozen sections of his crosssection, Fig. 3, and he evidently considers this exercise to be so straightforward (as do the Editors of

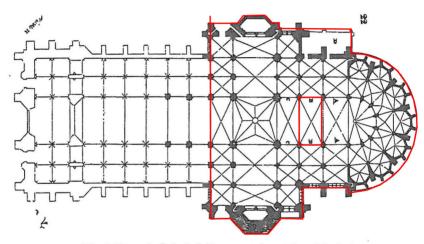


Fig. 1. Beauvais Cathedral: Reconstruction of the original plan by *Viollet-le-Duc*.

the Encyclopédie) that no explanations are given. As an example, Fig. 4 shows a redrawing of the conditions at the tas-de-charge, section CC in Benouville's analysis. On the tas-de-charge act: First of all, an inclined "poussée totale" from the rib vault, of magnitude 42,500 kg., i.e. 42.5 tonnes; secondly, a slightly inclined force of 76.2 tonnes² due to the weight of the material above the formeret, the parapet, and the great timber roof, combined with a small horizontal force contributed by the upper flying buttress, this total force of 76.2 tonnes having been determined from previous analysis (not given by Benouville) of a higher section; and, thirdly, two thrusts, of magnitudes 3 and 5 tonnes, contributed by the lower flying buttress. All these forces are summed in the force polygon of Fig. 4(b), and give a resultant of 111 tonnes transmitted on to the next cross-section to be considered, and eventually, of course, to the main nave pier and so to the foundations.

Benouville gives no account of how he determined the forces of 3 and 5 tonnes contributed by the lower flying buttress (nor, indeed, of why he splits up a single force into these two components,

¹ E. Corroyer, L'architecture Gothique, Paris (1891), p. 73.

 $^{^{2}}$ 1 tonne = 0.98 ton.

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although, as will be seen below, his train of thought on this matter is in fact quite clear). However, according to the principles of limit analysis applied to masonry construction, referred to above, there is no need for him to account for the values of these forces. What Benouville has done, in effect, is to calculate lines of thrust for the complete cross-section of Beauvais, for which equilibrium is satisfied everywhere, as exemplified by the force polygons. Further, these lines of thrust lie completely within the masonry (as they must); indeed, Benouville's solution involves thrusts lying in all cases very close to the centre lines of the members. No further test of stability is necessary. The safe theorem of limit analysis states that, if a thrust line *can be found* lying wholly within the masonry, then the structure is stable, and *there is no need to calculate the actual thrust line* (not that this could, in any case, be done with any confidence). Thus there is no need to determine the actual forces in

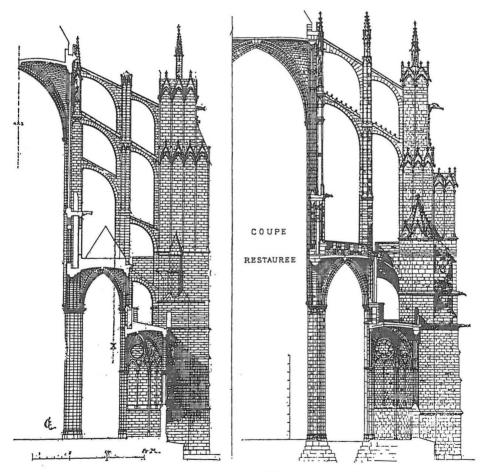


Fig. 2. Typical bay by Corroyer. Fig. 3. Coupe restorée by Benouville. Beauvais Cathedral: part cross-sections of the choir.

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the lower flying buttress; if Benouville can demonstrate (as he has) that forces of 3 and 5 tonnes will assure overall stability of the structure, then the structure is indeed stable under whatever forces actually occur in the flying buttress.

Benouville's solution astonished him by the magnitudes of the stresses that he found; the largest stress in the masonry he calculated as 13 kg./cm.²¹ This value may be compared with the crushing strength of a medium sandstone of from 150-400 kg./cm.2 (Ungewitter2), and confirms the generally low state of stress normally found in Gothic constructions. While Benouville cannot have had the

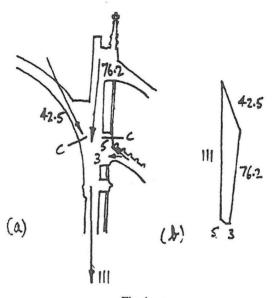


Fig. 4.

(a) Forces at the tas-de-charge of the main vault.

(b) Force polygon for these forces.

Beauvais Cathedral (Benouville).

comfortable assurance given by the limit theorems of structures that his analysis was correct, nevertheless his techniques had been used earlier for masonry, for example by Poleni in the analysis of the dome of St. Peter's,³ or by Yvon Villarceau in bridge design.⁴ In a sense, the engineer has always determined, as best he could, a "reasonable" set of forces in a structure on which to base his design; the limit theorems have now made respectable this pragmatic guesswork.

¹ Benouville, op. cit., "... or (nous-mêmes avons été surpris de ce résultat) sait-on à combien travaille la pierre la plus chargée de l'édifice? A treize kilogrammes par centimètre carré."

 ² G. G. Ungewitter, Lehrbuch der Gotischen Konstruktionen, 2 vol., Tauchnitz (1901), 142.
³ G. Poleni, Memorie istoriche della Gran Cupola del Tempio Vaticano, Padua (1748).

⁴ Yvon Villarceau, L'établissement des arches de pont, c.r. Acad. Sci., Paris, Mémoires présentés par divers savants, 12, 503 (1854). Yvon Villarceau's inverse design method for masonry arches consists in the assumption of a thrust line, to which the whole structure is then designed; he states quite clearly that elastic theory, leading to "correct" solutions, is not the tool for masonry arch design. For further discussion of this and of the dome of St. Peter's, Rome, see Heyman, "The Stone Skeleton."

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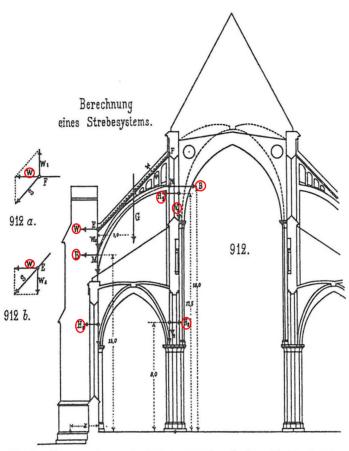
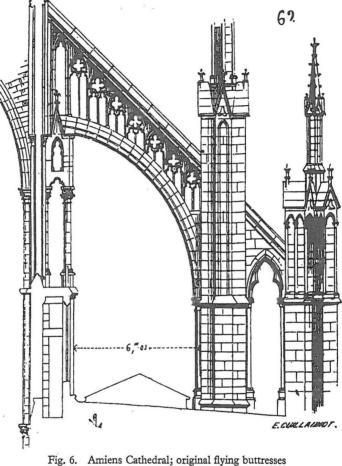


Fig. 5. Forces acting at a typical cross-section of a large Gothic church (Ungewitter).

Ungewitter quite frankly uses a final desired force distribution to calculate the magnitude of the buttress force. In Fig. 5, reproducing Ungewitter's Fig. 912, Plate 87, are shown the main forces acting on a typical cross-section. Considering first the main pier, Ungewitter determines the high vault thrust H_1 and vertical reaction V_1 , and similarly the aisle vault thrust H_2 and reaction V_2 . Making allowance for weights of walls, and so on, carried eccentrically by the pier, the question is then asked as to the magnitude of the buttress thrust B necessary for the total thrust line to pass precisely through the mid-point of the pier at the base.¹ If moments of the forces are taken about this midpoint, the value of B is then determined immediately (in Ungewitter's example, for which $H_1 = 3.24$ tonnes, the value of B is found to be 3.02 tonnes).

¹ Ungewitter, op. cit., p. 406. "Es soll zunächst berechnet werden, wie gross der Gegenschub des in 18 m Höhe anfallenden Strebebogens sein muss unter der Voraussetzung, dass der Druck unten durch den Mittelpunkt der Grundfläche des Mittelpfeilers geht." (Italics added.)

It will be seen in Fig. 5 that the flying buttress consists of two ribs, of which the lower is curved, both conducting the calculated thrust B to the main external buttress and also supporting the upper straight rib. Ungewitter assumes that this upper rib is normally free of load, but acts as a wind brace, necessary for the support of the upper part of the structure. The load W shown acting on this



(Viollet-le-Duc).

rib is computed from the wind pressure acting on the roof and part of the wall. The two ribs are only partially separated for the relatively low cathedral considered by Ungewitter; a complete separation was made at Amiens, Fig. 6,1 necessitated by the taller construction. The solution at Amiens was not satisfactory, almost certainly because of the tracery connection between the two ribs,²

Viollet-le-Duc, op. cit., I, 72, Fig. 62 (article Arc-boutant).
The probable buckling mode for the flying buttresses at Amiens is given in "The Stone Skeleton."

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and buttresses of this type now survive only at the chevet; the nave buttresses, originally of similar design, buckled, and were replaced in the fifteenth century.

Although the lower buttress at Beauvais is completely solid, Benouville evidently continued to think of it as a curved rib supporting a straight brace, and, to the horizontal force of 3 tonnes contributed by the curved rib he finds it necessary, to obtain the desired thrust distribution in the structure as a whole, to add an inclined thrust of 5 tonnes in the upper rib. As pointed out above, this is a completely legitimate procedure. While accepting the validity of Benouville's analysis, the examination below of the magnitudes of the forces acting on the fabric of Beauvais can help in understanding the particular form given to the structure, and specifically to the flying buttresses.

The main design requirement of a flying buttress system is to provide support both against the static vault thrust and also against the dynamic wind load, which has a resultant acting much higher up. An apparently completely satisfactory design was achieved by the use of two flying buttresses, as to the nave at Reims, Fig, 71; Fitchen² has pointed out that when such a double system is used, the lower buttress absorbs the vault thrust, and the upper acts as the wind brace. The separation of the two buttresses is dictated by the height of the parapet above the tas-de-charge; for Ungewitter's example, Fig. 5, for which the total height of the cathedral is about half that of Beauvais, the separation is small.

The height of the parapet is, in turn, related to the amount of doming given to the vaults by the designer. The parapet will be high if the vaults are virtually cylindrical, with level soffits, as at Beauvais, Fig. 3, or Reims, Fig. 7. If the vaults are strongly domed, as at Notre Dame, Paris, Fig. 8,³ then it may be possible to use a single flying buttress to counteract both the vault thrust and the wind forces.

Now the great flying buttresses at Paris span over two side aisles, and a buttress of these dimensions is unusual. It was more common to use an intermediate pier, as, indeed, was the case originally at Paris; the late twelfth century design is shown in Fig. 9.⁴ This design was destroyed, and rebuilt in its present form, et about the middle of the thirteenth century.⁵ Similarly, the *choir* at Reims, which has two side aisles on each side, uses intermediate piers in the buttressing system. And, of course, Beauvais has the intermediate buttresses in porte-à-faux.

The typical cross-section of Beauvais will be assumed to "look after" an axial length (9 m) of the structure, i.e. the axial <u>pier</u> spacing will be taken as 9 m. Thus with a choir width of (5 m), the plan area of a typical bay is 135 m², and the weight of a half-bay of vaulting may be estimated from Ungewitter⁵ as 36 tonnes. Ungewitter's Table also gives lines of action of the forces on the vault, and these are entered in the sketch of Fig. 10. The line of action of the horizontal thrust H, 6.5 m. below the crown of the vault, coincides exactly with the placing of the tas-de-charge at Beauvais. For equilibrium (H \times 6.5 = 36 \times 3.6), from which H is determined as 20 tonnes. The inclined reaction $R = \sqrt{(20)^2 + (36)^2} = 41.2$ tonnes agrees well enough with Benouville's "poussée totale" of 42.5; the angle of inclination of the thrust, $\tan^{-1} 36/20 = 61^\circ$, also agrees). Thus a horizontal thrust of 20 tonnes acts on the tas-de-charge; as Benouville demonstrated, not all of this need be transmitted by the flying buttress.

The outline of the lower flying buttress, spanning 4.5 m. between the tas-de-charge and the intermediate pier, and weighing about 5.0 tonnes, is shown in Fig. 11. The dotted line is the trace of the

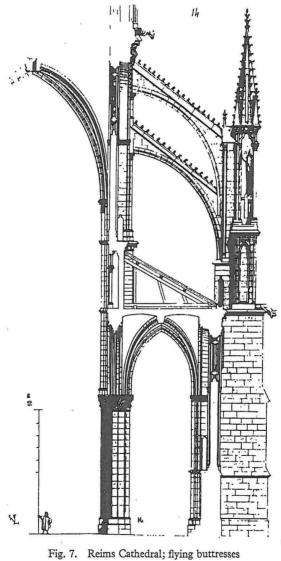
¹ Viollet-le-Duc, op. cit., 2, 318, Fig. 14 (article "Cathédrale").

² J. Fitchen, A comment on the function of the upper flying buttress in French Gothic Architecture, Gaz. Beaux-Arts, 45, 69 (1955).

³ Viollet-le Duc, op. cit., 1, 68, Fig. 59 (article "Arc-boutant"). ⁴ Idem, 2, 289, Fig. 2 (article "Cathédrale").

⁵ Idem, p. 288 ff.

Class IVc gives a unit weight of 530 kg/m², from which the weight of ⁶ Ungewitter, op. cit., p. 139, Tabelle 1. the half bay is determined as $(\frac{1}{2} \times 530 \times 135) = 35,800$ kg.



(Viollet-le-Duc).

passive line of thrust,¹ corresponding to the minimum possible horizontal thrust necessary for stability of the flying buttress (i.e. in the absence of a vault, the buttress would, in any case, thrust against the tas-de-charge with a force of minimum value, in this example, of 1·1 tonnes). Thus the upper flying buttress, normally unloaded, also pushes against the fabric with a minimum thrust of approximately the same magnitude.

The function of the intermediate buttress now becomes clearer. Had it been omitted, each flying

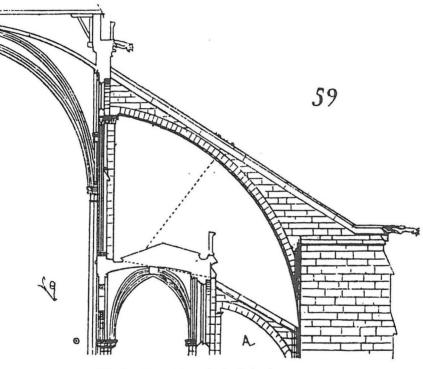


Fig. 8. Notre Dame, Paris; flying buttresses (*Viollet-le-Duc*).

buttress would have had to span 9.5 m. instead of the 4.5 m. of Fig. 11. Rules of design were numerical rules; the buttress of 9.5 span, would, almost certainly, have been geometrically similar to that of 4.5 m. span, i.e. all its proportions would have been increased in the same ratio, 9.5/4.5. Scaling up Fig. 11 in this *linear* ratio, all the forces should be increased by the factor $(9.5/4.5)^1$, i.e. the larger flying buttress would weigh some 47 tonnes, and the minimum passive buttress thrust becomes 10.3 tonnes. The lower buttress would still be satisfactory, since it would transmit some proportion, between 10.3 and 20 tonnes, of the full vault thrust of 20 tonnes.

The upper flying buttress, however, would lean against the flat wall of the parapet, also with a mini-

¹ For the calculation of such a line, see "The Stone Skeleton."

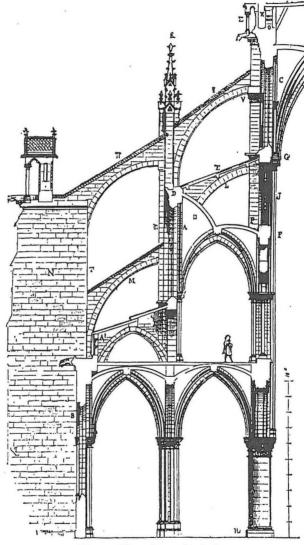
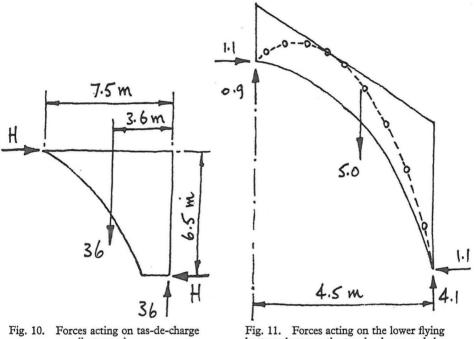


Fig. 9. Notre Dame, Paris; late twelfth-century design (Viollet-le-Duc).

mum force of about 10 tonnes, and some distress could be caused to the masonry in this region.¹ The introduction of the intermediate pier causes a dramatic reduction in the values of the passive thrusts of the flying buttresses, which are turned into elegant light props, hardly pressing against the fabric when at rest, but capable of enormous thrusts if called upon to resist wind.

The relative narrowness of the side aisles at Beauvais helps to lead to this elegance. The flying buttresses over the single nave aisle at Reims, Fig. 7, have a span of 7.5 m., and the passive thrust of each is therefore about $(1\cdot 1)(7\cdot 5/4\cdot 5)^3$, or about 5 tonnes. The masonry as a whole at Reims is, of



(in tonnes).

buttress between the tas-de-charge and the intermediate pier.

Beauvais Cathedral.

course, very heavy; the vaults are about 60 cm. thick, compared with the more usual 20 cm., and the parapets are massive. The single great buttresses at Paris span about 11 m.; similar rules of proportions would give a passive thrust of 16 tonnes, which agrees well with the value given by a direct calculation.²

It is of interest to calculate wind loads. Fitchen³ estimates that the upper flying buttress at Reims is subjected to a maximum wind load of 15 tons. At Beauvais, if the great roof presents an area (per masonry bay) of $12 \times 9 = 108 \text{ m}^2$ to the wind, and the unit wind pressure is say 150 kg./m.²,

¹ No distress would be caused, however, at the chevet, since the buttress no longer thrusts against a flat surface; the curvature of the plan allows a "triangulation" of the forces. ² 12 to 15 tons is given in "The Stone Skeleton."

³ Fitchen, op. cit.

then the wind force on the roof is about 16 tonnes. In addition, both flying buttresses must resist the pressure of the wind on the vertical wall of the choir.

The "porte-à-faux" of the intermediate buttresses has not yet been discussed. Although they are not placed vertically over the supporting piers, there is in fact no sense in which they are falsely carried. Had they been placed farther inwards, say to over the centre of the aisle, Viollet-le-Duc would have agreed that, with proper design, the aisle arch could have carried them. Indeed, the question should be inverted: Can any possible mechanism of collapse be thought of for the intermediate buttress? The answer, just as for the flying buttresses of Beauvais, is no; only collapse of the outer main buttresses will permit consequential collapse of the intermediate buttresses.

Thus the fabric of Beauvais in 1272 seems to have been, in the large, designed almost perfectly to fulfil its function. No mention has been made of the main external buttresses, but Benouville's analysis shows them to be very stable indeed. Similarly, no attention has been paid to the design of individual yault severies; it is extremely unlikely that individual panels would have fallen, and that was not the sort of catastrophe that evidently occurred in 1284. The possibility therefore cannot be overlooked that the collapse began with some trivial accident and spread thence to the whole of the fabric. Viollet-le-Duc indeed gives such an explanation.

His cross-section is shown in Fig. 12¹; this cross-section is not of the typical bay, but is taken at the chevet, where the ground plan, Fig. 1, permits the main external buttresses to be placed closer in. A perspective sketch is given in Fig. 13.²

Viollet-le-Duc considers that the slender twin columns A (Fig. 13) failed.³ The mortar, slowly drying in the adjacent pier B, shrank (perhaps because the work was too hastily done, as Benouville believes), and more and more load was thrown onto the twin columns until they eventually fractured. As a consequence, the lintel L broke, and the great block M, the tas-de-charge, loaded by the gigantic statue N, was no longer supported. Viollet-le-Duc then considers that gross deformation occurred, which is plausible enough. He suggests that the block M slid out. It is more likely, however, that the block M would tilt outwards and so drive the line of thrust (shown broken in Fig. 11) outside the section of the flying buttress which would then collapse. There would be nothing to counteract the vault thrust; the vault would then collapse in that bay, almost certainly completely across the choir, so that at least one complete bay of vaulting would fall. The collapse would be likely to spread, since each bay of vaulting buttresses the next in an axial direction.

The typical Gothic structure is, in fact, an example of an assemblage of structural elements acting one on another to assure complete equilibrium of the whole. The structure can accomodate a wide range of forces, but, take away one portion, and all the rest is likely to fall. Without trivial accidents, however, and assuming no Acts of God, the complete structure was so stable that it would remain for centuries.

Guessing the weight of Jean Vast's tower as 2,000 tonnes, and taking each crossing pier as of area 3 m.² the tower would have imposed an extra stress of some 16 kg./cm.² on these piers. This is very small, and again there is no question of the strength of the material governing the behaviour.

¹ Viollet-le Duc, op. cit., 4, 178, Fig. 101 (article "Construction").

² Idem. p. 181, Fig. 101 ter. ³ Idem, p. 180 f. "Il est certain cependant que cet énorme édifice aurait conservé une parfait stabilité, si l'architecte eût posé les colonnettes jumelles au-dessus du triforium plus fortes et plus résistantes, s'il eût pu les faire de fonte, par exemple. Les désordres qui se sont manifestés dans la construction sont venus tous de là; ces colon-nettes, trop grêles, se sont brisées, car elles ne pouvaient résister à la charge qui se reporta sur elles lorsque les piles intérieures vinrent à tasser par suite de la dessiccation des mortiers. Se brisant, les linteaux L cassèrent (fig. 101); les gros blocs M, en bascule, s'appuyèrent trop fortement sur la tête du premier arc-boutant, celui-ci se déforma, et la voûte suivant le mouvement, la pression sur ces arcs-boutants fut telle qu'ils se chantournèrent presque tous; leur action devint nulle, par suite les arcs-boutants supérieurs lâchèrent un peu, puisque la voûte ne pressait plus sur eux. L'équilibre était rompu."

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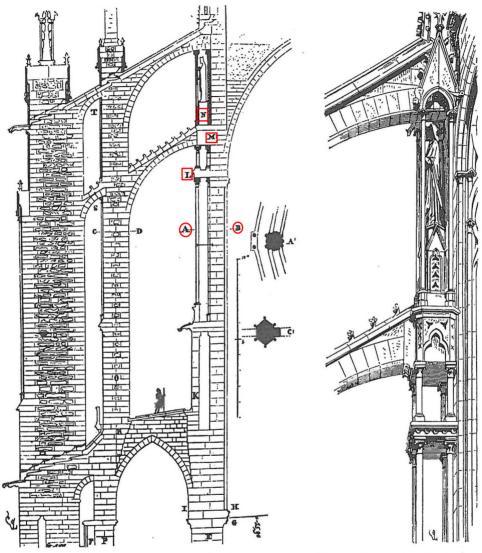
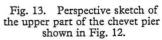


Fig. 12. Part cross-section at the chevet.



Beauvais Cathedral (Viollet-le-Duc).

There seems little doubt, however, that had the advice of the King's masons been taken in 1571, and the crossing piers braced, the tower might be standing today. It seems that the structure, from 1569, when the tower was completed, until 1573, when it fell, was never truly in equilibrium. Desjardins reports¹ numerous small movements, and Leblond² reports fractures occuring, during these 4 years.³

The structural system of a massive tower supported by four unbraced piers would be liable to "drift", the movement restrained by tensile and shearing stresses developed in the mortar, and by possible interlocking of stones. Eventually, however, the columns would have been pushed so far out of true as to be useless. The Chapter was right to call a halt in 1605; "le temps n'était plus à bâtir des cathédrales."

DISCUSSION

In reply to a comment that, when the erection of additional columns doubled the number of arches, no buttresses were built to the new intermediate piers, Dr HEYMAN replied that 'perhaps light could be thrown on this by referring to Paris. Great flying buttresses now occur at every pier in Paris, but there was a celebrated dispute just before the war—never really settled—as to whether the Cathedral had been built with a flying buttress at every bay or only every other bay—quite clearly the main thrust occurred at every other bay. It is possible, Dr Heyman thought from the evidence at Beauvais, that Paris could have been built with alternate buttresses omitted and that they were added later. It is practically certain that the architect at Beauvais would have seen the Paris construction.

Mr. R. J. MAINSTONE said that Dr. Heyman had set alongside the recorded fact, that the high vaults of Beauvais had stood for some 12 years, a demonstration that equilibrium between thrust and counterthrust was possible (barring gross deformations) and had concluded from the demonstration that the subsequent collapse must be attributed to some detail of construction rather than to a major fault of design. Mr. Mainstone accepted the demonstration (granted that Benouville's reconstruction of the original cross-section was correct), but he questioned the conclusion.

He accepted also Dr. Heyman's assertion that Gothic rules of design were numerical ones (taking this to include also geometrical ones), but questioned the interpretation placed on these rules. It seemed a great over-simplification to suggest that they resulted simply in a direct geometrical scaling of similar elements from one structure to another. Indeed both the present paper and the earlier one on "The Stone Skeleton" contained ample evidence that this was not the case. The comparative heaviness of the masonry at Reims had, for instance, been commented on, and the proportions at Beauvais seemed, on the contrary, to be significantly more slender than usual elsewhere. One almost had the impression that Reims was a lower structure than it should have been but that Beauvais was a higher one than originally intended.

The relevance of this to the possible cause of the collapse lay in the behaviour of the structure as the mortar of the piers slowly dried out. Though mean stress levels were low, non-axiality of the thrust would lead to progressive deformations, and the more slender the piers the greater the likelihood that the deformations would increase to the point where equilibrium was no longer ensured. An element of doubt about the precise form of the vaults and supporting structure above triforium level prior to the collapse made it undesirable to be dogmatic. The spread at the springings of the reconstructed vault was, however, measured in 1903 by William Goodyear. He found it to be about one metre and to exceed that at any other major cathedral. It is reasonable to assume that, immediately prior to the collapse of the original vault, it was at least similar to this and such a gross deformation

¹ Desjardins, op. cit., p. 92 f.

² Leblond, op. cit., p. 30.

³ A very similar problem was encountered at Wells in the early fourteenth century. The designer, William Joy, successfully braced the crossing piers with the famous "strainer arches."

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seems, by itself, to be sufficient to bring the slender upper piers of Benouville's cross section to the point of bucking and to lead to the collapse of the vault with or without the actual prior buckling of the piers. The manner of reconstruction, with its emphasis on stiffening the piers and intercalating others is, moreover, entirely consistent with this interpretation.

Mr. Mainstone therefore preferred to attribute the collapse primarily to an excessive slenderness of the upper parts of the main piers in relation to their manner of construction and to consequent excessive deformations under eccentric load. He strongly suspected, though, that a close examination of the present fabric to determine the precise extent and manner of the reconstruction would provide the necessary basis for a more definitive conclusion and regretted that the paper was silent about this. Could Dr. Heyman say what evidence he had been able to find?

Professor A. W. SKEMPTON said heremembered very clearly an expedition to various French cathedrals during which he received an overwhelming impression that the proportions of Beauvais were markedly different from the other structures. If the major failure at Beauvais began as a trivial accident why did the people concerned decide to double the number of piers when rebuilding? From the manner of the reconstruction it would seem evident that originally something must have been wrong with the piers. Moreover, Professor Skempton did not entirely agree with the idea that the stresses were very low. The piers were probably built with a good masonry casing having a rubble core. Recently he had seen the remedial works in progress on the piers at Winchester. No doubt here the average stresses would also be apparently quite small, but nevertheless many of the piers of the Middle Gothic reconstruction were severely cracked, with fissures several inches wide running almost from top to bottom. Given the extraordinary proportions at Beauvais, and taking into account the fact that the piers were doubled in number in the rebuilding, there could be little doubt that overstressing of the piers was a principal cause of the disaster.

Professor Skempton was intrigued by Dr. Heyman's reference to the evidence for using large models. One knows, of course, that models were built, the equivalent of present-day architectural models, to show the client what was intended, but he had never been convinced that structural models were used in medieval times.

Dr. HEYMAN said that there was no clear account of how the collapse of 1284 did occur; there are conflicting accounts—none contemporary. One report says that external buttresses failed, but this is not supported by others. There is no evidence to put against Mr. Mainstone's view that the vaults fell without bringing down the external buttresses. As to the rubble filling, at least in the sixteenth century they were very much alive to the stress on the main crossing piers and the King's masons made the specific remark that these piers were solid all the way through. The evidence for the use of models was quite strong, and several examples are given by Frankl in *The Gothic*.

Mr. GRANT stated that at Chartres the original flying buttresses had consisted of two slightly curved struts held apart by a series of pillars rather like the spokes of a wheel. The expertise of 1313 recommended the addition of a third strut above the other two. At Amiens in 1497 an expertise recommended a third strut below the existing openwork buttress. At the same time an iron band was fixed round the whole cathedral to stop the spread at the head of the columns at the crossing which was leaning outwards.

Dr. Heyman replied that the slow distortion and spread of the great timber roof might have much to do with the eventual requirement for a third buttress at parapet level.

Mr. MAINSTONE said that he found it very difficult to visualise how the tas-de-charge M could slide out. According to Viollet-le-Duc's drawing it was built substantially into the main pier. Also it was under compression from both sides; the vault pushing it out from one side and the buttress arch pushing it in from the other. Even it if cracked it was in no different situation from that of any

other voussoir of the buttress arch except that, being weighted from above, it could not so easily ride up the pier as this inclined outward.

Dr. HEYMAN commented that he thought the tas-de-charge could have crushed or cracked and dropped out rather than slid out with very little deformation on the main buttresses. He had no evidence either way about the original structure having been altered between 1250 and 1350; and there is no evidence that the structure was built other than in the form we see now; apart from the intercalated piers and reconstruction of the vaults themselves.

Mr. R. J. M. SUTHERLAND asked if it was known for certain that the choir which collapsed in 1284 was designed exactly as it is now; was the amount of buttressing as great as now; or was it rather like the Lincoln Chapter House where flying buttresses were added nearly ahundred years after the Chapter House was built. He would like to support Professor Skempton in the point about stresses. The stresses could become heavily concentrated and cause splitting as in the anchorage zones of prestressed beams. The strength of an individual brick or stone gives little indication of the strength of the wall in such circumstances. On the question of how the failure of the choir went: if the tas-de-charge did slide out then the buttresses would have had to move outwards in order to have let the tas-de-charge out, and if that happened Mr. Sutherland imagined that the buttresses could have remained standing even after the arcade had fallen. Alternatively it could have been one of those concentrated-load failures in which a piece crushed, split and came out in bits.

Mr. H. CLAUSEN said he had filmed most of the stained glass of the French cathedrals and he would like to support Professor Skempton's impression that Beauvais presented completely different proportions compared with the other great French cathedrals. He commented that the builders of Beauvais were always short of money. They got permission from the Pope to sell indulgences, and did so, to raise the money to build the tower. Why did they not go for the nave which would have provided the buttressing for the crossing? Isn't this the only case where the tower was built before the nave? In most cathedrals the nave and the choir and the transept were built first, and then the tower added after the base structure was more or less complete.

Mr. IAN DAVIDSON also emphasized the lack of buttresses or other means to resist longitudinal thrust. The obvious remedy was to put in interpolated piers thus halving the span and reducing the unbalanced longitudinal thrust.

Dr. HEYMAN agreed that the unbalanced longitudinal thrust, although small, could well have contributed to the collapse of the tower in 1573, but did not think that this effect explained the main collapse of the high vaults in 1284.

Mr. M. H. L. STANDEN asked whether the iron ties and staves at the top, had they been there for hundreds of years, would not have rotted away, or grown very much larger by corrosion so lifting and disrupting the masonry.

Dr. HEYMAN said that the iron ties that could be seen were said to be fourteenth century; he thought they were not there originally. He had not been up and did not know the state of the stonework at that level. In good building iron would be sealed with lead; this would reduce the risk of rusting.

Mr. E. W. H. GIFFORD stated that at Salisbury the masonry tower contains a great deal of iron (from about 1360 and later) some of it enclosed in lead, some not. The stone, as wet inside as out and more continuously so, gives it no protection. In some places it has remained in very good order; in others it is giving trouble.

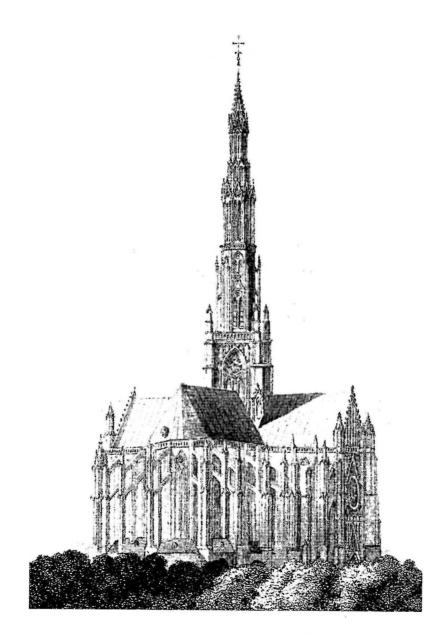
In proposing the vote of thanks Dr. S. B. Hamilton said that Dr. Heyman had given a most interesting address, on a subject of great interest to some, but one on which the Society did not hear much at its meetings. The cause of failure at Beauvais may have been composite. Dr. Hamilton thought

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that the possibility of foundation movement could not be ruled out altogether. In a building of that size there is almost certain to have been some differential settlement. Allowance was apparently made for wind but Dr. Hamilton wondered if they realised what the movement from wind on a building of that size would be. Dr. Hamilton said there would be both plastic and elastic distortion. Some of the piers were heavily loaded on one side and, as we know, they do bend; for instance in Westminster Abbey there is a difference of some inches between the distance apart of the main columns of the crossing at capital level and at floor level which is quite obvious without plumb lines or instruments. This type of distortion can be seen in nearly all tall Gothic buildings. There is also the end thrust of the arcade down the length of the building, as mentioned by Mr. Davidson. We know that this caused collapse at Hereford Cathedral, Malvern Abbey and several other well-known buildings, and that it could quite well have contributed to the trouble at Beauvais. One could imagine the building settling, thrusting out at the ends, spreading crosswise and then, twelve years after building just being unable to stand any more movement, coming down with a rush. If Dr. Heyman hadn't cleared up all the mystery he had given the audience a great deal to think about that should help us to realise the cumulative movement that goes on in many of these important and most interesting Gothic structures.

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Plate V



Beauvais Cathedral as completed in 1569. J. M. Fugère sculp. Frontispiece from Desjardins